

its designed value by several times the separation between *réseau* and plate ; and in the second place upon the actual focal length as affected by temperature from day to day.

It appears, then, that we lose nothing by employing light diverging from a source at the focal distance instead of the more or less parallel light from the same source in the focus of a collimator. And in one respect we gain. Suppose the plate is not flat. There will be irregular distortion in the field, since the stars are projected from the centre of the objective. If we put on the *réseau* with parallel light, there is no distortion of its image on the plate ; if with light from a source at a distance from the plate equal to the focal length, the distortions of the *réseau* image are approximately the same as the distortions of the neighbouring stars, unless the irregularities in the plate are very abrupt. We have thus a method of avoiding to a large extent the accidental errors due to curvature of the plate.

My conclusions are, then, that it is advisable to increase the width of the ruled *réseau* lines to 0^{mm}·02, and to impress them on the plate by light from a source placed without any collimator at a distance equal to the focal length of the photographic objective.

Cambridge Observatory :
1899 June 6.

On the formulæ of reduction to the meridian of the observed zenith distances of stars. By A. E. Young.

The author being unable to reconcile the formulæ given in the "Handbook of Professional Instructions for the Trigonometrical Branch, Survey of India Department," p. 177, for the correction to the observed zenith distance of a star for instrumental error, with those given by Chauvenet in his "Spherical and Practical Astronomy," 5th ed. vol. ii. pp. 290–291, for the meridian circle, was led to an examination of the latter, from which it appears that the discrepancies were caused in the first place by some small terms having been neglected by Chauvenet in forming the expansion for the correction, and in the second to his having assumed that the correction to what he calls the instrumental declination is the same as the correction to the instrumental zenith distance of the star. At any rate, the latter assumption is made in the example he gives. These assumptions, which involve neglecting terms depending on n only, will hardly affect observations made with the meridian circle, in which n is usually very small ; but in the case of a portable altazimuth or zenith telescope they may have an appreciable effect, and it will be inter-

esting to examine them and, at the same time, show how to deduce the survey of India formulæ from Chauvenet's.

The instrumental declination δ' is really the difference between the readings on the pole and on the star subtracted from 90° . In practice, however, the reading on the pole cannot be obtained, so that a correction must be computed for it and combined with the correction to the reading on the star. For let

$$z_p = \text{observed reading on pole}$$

$$z_s = \quad \quad \quad \text{,,} \quad \quad \quad \text{,,} \quad \text{star}$$

and let Δp , Δs be the corrections to these for instrumental error, so that

$$z_p + \Delta p = \text{true Z.D. of pole}$$

$$z_s + \Delta s = \quad \quad \quad \text{,,} \quad \quad \quad \text{,,} \quad \text{star.}$$

Then, in the case of a star north of the zenith,

$$\delta' = 90^\circ - (z_p - z_s)$$

$$\delta = 90^\circ - \{(z_p + \Delta p) - (z_s + \Delta s)\}$$

$$\therefore \delta' - \delta = \Delta p - \Delta s$$

$$\therefore \Delta s = \Delta p - (\delta' - \delta)$$

and, in the case of a star south of the zenith,

$$\Delta's = (\delta' - \delta) - \Delta p = -\Delta s.$$

Taking the last of Chauvenet's three equations (191), we have

$$[\cos \delta \cos (\tau - m) = \cos c \cos \delta']$$

Put $\delta' = \delta + h$ and expand, then

$$\cos \delta \cos (\tau - m) = \cos c (\cos \delta - h \sin \delta)$$

or

$$h = \cot \delta \left\{ 1 - \cos \frac{(\tau - m)}{\cos c} \right\}.$$

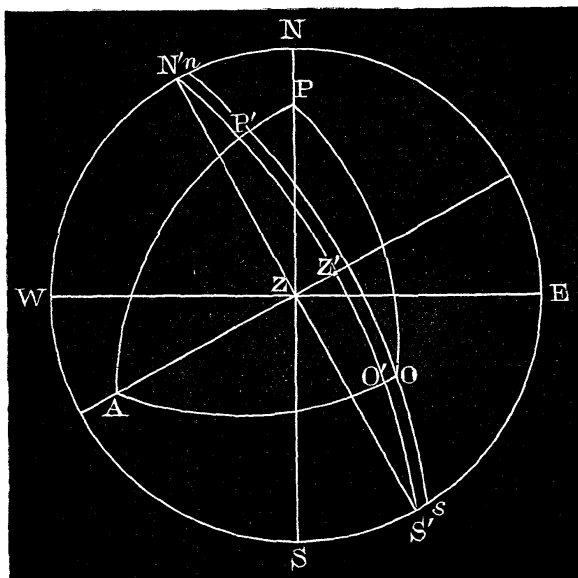
Expanding $\cos (\tau - m)$ and $\cos c$ to two terms,

$$h = \delta' - \delta = \frac{\cot \delta}{2} \{(\tau - m)^2 - c^2\} \dots \dots \dots (A)$$

If we insert for c its value $(\tau - m) \cos \delta - n \sin \delta$, this reduces to

$$\begin{aligned} \delta' - \delta &= \frac{\cot \delta}{2} \{(\tau - m)^2 - (\tau - m)^2 \cos^2 \delta + 2n(\tau - m) \sin \delta \cos \delta - n^2 \sin^2 \delta\} \\ &= \frac{(\tau - m)^2}{4} \sin^2 2\delta + n(\tau - m) \cos^2 \delta - \frac{n^2}{4} \sin 2\delta. \end{aligned}$$

This differs from the first term of Chauvenet's expansion (193) by the second and third terms, which therefore show the error of that expansion.



Having now found the correct expression for $\delta' - \delta$, it remains to find Δp . Using Chauvenet's Fig. 48, $\Delta p = ZP - Z'P'$. By spherical trigonometry, since

$$AP' = 90^\circ \text{ and } AO' = 90^\circ \\ \cos Z'P' = \cos PAZ,$$

we also have

$$PA = 90^\circ - n; \quad AZ = 90^\circ - b. \\ \therefore \cos ZP = \sin b \sin n + \cos b \cos n \cos PAZ \\ = \sin b \sin n + \cos b \cos n \cos P'Z';$$

or expanding and putting $Z'P' = ZP - \Delta p$,

$$\cos ZP = bn + \left(1 - \frac{b^2}{2} - \frac{n^2}{2}\right) (\cos ZP + \Delta p \sin ZP)$$

but

$$ZP = 90^\circ - \phi. \\ \therefore \sin \phi = bn + \left\{1 - \frac{1}{2}(b^2 + n^2)\right\} (\sin \phi + \Delta p \cos \phi). \\ \therefore \Delta p = \frac{\tan \phi}{2} (b^2 + n^2) - \frac{bn}{\cos \phi} \quad \dots \dots \dots (B)$$

$$\therefore \Delta s = \frac{\tan \phi}{2} (b^2 + n^2) - \frac{bn}{\cos \phi} - \frac{\cot \delta}{2} \{(\tau - m)^2 - c^2\} \quad \dots \dots (C)$$

for north stars ;

$$= \frac{\cot \delta}{2} \{(\tau - m)^2 - c^2\} + \frac{bn}{\cos \phi} - \frac{\tan \phi}{2} (b^2 + n^2)$$

for south stars.

This is the complete expression for the correction to an observed zenith distance for instrumental error, and by substituting in it the values of a , b , and c , the numerical value will be found.

Correction for Collimation Error c.—Suppose $a=0$ and $b=0$ then

$$n=0 \text{ and } \tau-m=c \sec \delta$$

$$\therefore \Delta s = -\frac{\cot \delta}{2} c^2 \frac{(1 - \cos^2 \delta)}{\cos^2 \delta} = -\frac{c^2}{2} \tan \delta$$

for a north star, and $+\frac{1}{2}c^2 \tan \delta$ for a south star. Of course, if the star is south of equator δ will be negative, and the sign of the correction will be reversed.

Correction for Azimuthal Deviation.—Put $b=0$ and $c=0$, then

$$n = -a \cos \phi, \tau-m = -a \cos \phi \tan \delta$$

$$\Delta s = \frac{\tan \phi}{2} \cdot a^2 \cos^2 \phi - \frac{\cot \delta}{2} (a^2 \cos^2 \phi \tan^2 \delta)$$

$$= \frac{a^2}{2} (\sin \phi \cos \phi - \cos^2 \phi \tan \delta)$$

$$= \frac{a^2 \cos \phi \sin (\phi - \delta)}{2 \cos \delta}$$

$$= -\frac{a^2 \cos \phi \sin z}{2 \cos \delta}$$

for both north and south stars.

Correction for Dislevelment of the Transit Axis.—Put $a=0$ and $c=0$, then

$$n = b \sin \phi, \tau-m = b \sin \phi \tan \delta,$$

$$\Delta s = \frac{b^2 \tan \phi}{2} (1 + \sin^2 \phi) - \frac{b^2 \sin \phi}{\cos \phi} - \frac{\cot \delta}{2} b^2 \sin^2 \phi \tan^2 \delta$$

$$= -\frac{b^2 \sin \phi}{2 \cos \phi} (1 - \sin^2 \phi) - \frac{b^2 \sin^2 \phi \tan \delta}{2}$$

$$= -\frac{b^2 \sin \phi}{2 \cos \delta} (\cos \phi \cos \delta + \sin \phi \sin \delta)$$

$$= -\frac{b^2 \sin \phi \cos (\phi - \delta)}{2 \cos \delta}$$

$$= -\frac{b^2 \sin \phi \cos z}{2 \cos \delta} \text{ for north stars}$$

$$= +\frac{b^2 \sin \phi \cos z}{2 \cos \delta} \text{ for south stars.}$$

The above formulæ agree with those of the “Survey of India Handbook.” They can also be deduced from Mayer’s formula for the correction to an observed transit by substituting the

value of τ corresponding to a , b , and c in the formula for circum-meridian altitudes—viz. $\frac{\tau^2}{2} \frac{\cos \phi \cos \delta}{\sin z}$, which is always subtractive from the observed zenith distance, and then correcting the result in the case of b and c by the corrections* to an observed zenith distance for an instrument out of level and collimation, viz.

$$\frac{b^2 \cot z}{2} \text{ and } \frac{c^2 \cot z}{2},$$

which are always additive to the observed zenith distance. For instance,

Collimation Error.— $\tau = c \sec \delta$.

Correction

$$\begin{aligned} &= \frac{c^2}{2} \cot z - \frac{c^2 \sec^2 \delta \cos \phi \cos \delta}{2 \sin z} \\ \Delta s &= \frac{c^2}{2 \sin z} \left(\cos z - \frac{\cos \phi}{\cos \delta} \right) = \frac{c^2}{2 \sin z \cos \delta} (\cos z \cos \delta - \cos \phi). \end{aligned}$$

If the star is N., $\phi = z - \delta$,

$$\begin{aligned} \Delta s &= \frac{c^2}{2 \sin z \cos \delta} (\cos z \cos \delta - \cos z \cos \delta - \sin z \sin \delta) \\ &= -\frac{c^2 \tan \delta}{2} \end{aligned}$$

as before.

Dislevelment Error.— $\tau = b \frac{\cos z}{\cos \delta}$,

$$\begin{aligned} \Delta s &= \frac{b^2 \cot z}{2} - \frac{b^2 \cos^2 z}{2 \cos^2 \delta} \cdot \frac{\cos \phi \cos \delta}{\sin z} \\ &= \frac{b^2 \cos z}{2 \sin z \cos \delta} (\cos \delta - \cos z \cos \phi). \end{aligned}$$

If the star be N., $z = \delta - \phi$ or $\delta = z + \phi$,

$$\begin{aligned} \Delta s &= \frac{b^2 \cos z}{2 \sin z \cos \delta} (\cos z \cos \phi - \sin z \sin \phi - \cos z \cos \phi) \\ &= -\frac{b^2 \sin \phi \cos z}{2 \cos \delta} \end{aligned}$$

as before.

The correction for azimuth deviation follows at once by substituting

$$\tau = \frac{a \sin (\phi - \delta)}{\cos \delta}.$$

When the three causes of error, a , b , and c , occur together the complete formula (C) should be used. Very erroneous results

* See Chauvenet, vol. ii. p. 327.

may be obtained by combining the formulæ for a , b , and c separately, as the complete expression contains the products of a , b , and c as well as their squares. The following example shows this (it must be remembered that the above expressions are to be multiplied by $\sin 1''$ if a , b , and c are given in seconds of arc) :

$$\begin{aligned} a &= +112'', \quad b = +25'', \quad c = +30'' \\ \phi &= +25^\circ, \quad \delta = +35^\circ, \quad z = 10^\circ. \\ \text{Correction for } c \text{ alone} &= -0.00153 \\ \text{'' '' } a \text{ ''} &= -0.00584 \\ \text{'' '' } b \text{ ''} &= -0.00077 \end{aligned}$$

whereas correction for a , b , and c all occurring together $= +0''.0228$, which is three times as great and of opposite sign to the sum of the three corrections taken separately. It must also be remembered that a , b , and c are considered in this formula as being positive when they all tend to make the transit of a south star occur too early.

Taiping, Perak: April 1899.

Small Nebulæ discovered with the Crossley Reflector of the Lick Observatory.

By James E. Keeler, D.Sc., Director.

The following small nebulæ were found on two plates exposed with the Crossley reflector, for three and four hours respectively, for the purpose of photographing the spiral nebulæ $M\ 51$ in *Canes Venatici*. Assuming the position of the nucleus of the great spiral to be R.A. $= 13^h\ 25^m\ 39^s$, Decl. $= +47^\circ\ 42'.6$, as given by Roberts, the positions of the small nebulæ for 1900 are as below :

No.	R.A. h m s	Decl. ° ' "	Description.
1	13 23 02	+47 23'3	Round; diam. = 0'.2.
2	24 17	26.5	Spindle-shaped; length = 1'.9.
3	26 05	49.7	Very narrow; length = 0'.6.
4	26 14	45.5	Round; diam. = 0'.2; central condensation.
5	27 07	41.6	Round, diffuse; diam. = 0'.3.
6	27 19	18.3	Round; diam. = 0'.15.
7	27 33	19.8	Slightly elongated; major axis = 0'.2.

No. 2 is long and narrow, with a bright, somewhat irregular axis. No. 3 and No. 4 are close to the great spiral, but apparently not connected with it. The former is recognisable on